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LETTER TO THE EDITOR

Aharonov–Casher effect and geometrical phases

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Abstract. The phaseshift experienced by a neutral particle with a magnetic moment as it encircles an infinite line of charge (Aharonov-Casher effect) is derived as a special case of geometrical phases, i.e. the standard Berry phase and the gauge-invariant Yang phase.

It is well known that the view that the vector potential in electrodynamics does not produce any observable physical effect was challenged for the first time by Aharonov and Bohm [1] (AB). Indeed, the famous effect named after them states that a charged particle experiences a phaseshift as it encircles an infinitely long solenoid (even though there is no magnetic field, but only a vector potential, in the region where it travels). The many doubts which have arisen since then about the real existence of this effect were definitely removed a few years ago by the conclusive experiment by Tonomura *et al* [2].

In 1984, Aharonov and Casher [3] (AC) suggested a 'dual' of the AB effect: they predicted that a neutral particle with a magnetic moment experiences a phaseshift as it encircles an infinite line of charge (the particle spin being parallel to the charged line). Theoretical contributions to the understanding of the AC effect have been subsequently given by Aharonov *et al* [4], Goldhaber [5] and Anandan [6]. The first experimental evidence of the AC effect has been given very recently [7].

Both the AB and AC effects have a distinctively topological nature, because the generation of relative phases in the wavefunctions of the corresponding particles is only related to the fact that they travel in a non-simply-connected, force-free region (i.e. their paths encircle a singularity of the field).

Another class of geometrical phases arising, under suitable conditions, in the evolution of quantum systems is that of Berry's phases [8, 9]. Although, in Berry's original paper, adiabatic and cyclic evolution of the system are needed for the generation of a geometrical phase in the system wavefunction, subsequent studies have shown that both these conditions can be relaxed [10–12].

As it is easy to realize, there is a strict connection between AB and AC effects and Berry's phases. Indeed, Berry himself was the first to show, in the original paper of 1984, that the AB phase is just a special case of his geometrical phase [8]. Another proof was then given by Aharonov and Anandan [10].

The aim of this paper is to show that—as expected—the Aharonov-Casher phase is also just a special case of Berry's phases. Our proofs will make use first of the original form of Berry's phase, and then of the gauge-invariant Yang phase [13, 14] (which reduces to the Aharonov-Anandan phase for cyclic, but non-adiabatic, processes).

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Consider a quantum system consisting of a neutral particle with magnetic dipole moment $\boldsymbol{\mu}$, parallel to an infinite line of charge.

Assume that the particle occurs in a plane perpendicular to the charged line, and that it is confined to a box situated at $\boldsymbol{R}$. If $\hat{\boldsymbol{r}}$ is the position operator of the particle and $\hat{\boldsymbol{p}}$ its conjugate momentum, the (non-relativistic) particle Hamiltonian can be written as [3] ($\hbar = c = 1$)

$$\hat{H}_{NR} = \frac{1}{2m}(\hat{p} - E \times \boldsymbol{\mu})^2 \tag{1}$$

where $\boldsymbol{E}(\hat{\boldsymbol{r}})$ is the electric field generated by the infinite charge distribution (supposed to bear a charge linear density λ). In the absence of the electric field, the wavefunctions have the form $\Psi_n(\boldsymbol{r} - \boldsymbol{R})$ (with energies ε_n independent of $\boldsymbol{R}$). Then, the eigenfunction $|n(\boldsymbol{R})\rangle$ of the Hamiltonian (1) can be written as

$$\langle \boldsymbol{r}|n(\boldsymbol{R})\rangle = \exp\left\{ \mathrm{i} \int_{\boldsymbol{R}}^{\boldsymbol{r}} \mathrm{d}\boldsymbol{r}' \cdot (\boldsymbol{E}(\boldsymbol{r}') \times \boldsymbol{\mu}) \right\}. \tag{2}$$

If the box is transported (not necessarily adiabatically) around a circuit C, the particle wavefunction acquires a Berry phase given by [8]

$$\gamma_n(C) = \mathrm{i} \oint_C \langle n(\boldsymbol{R}')|\nabla_{\boldsymbol{R}} n(\boldsymbol{R}')\rangle \,\mathrm{d}\boldsymbol{R}'. \tag{3}$$

We have therefore

$$\begin{aligned} &\langle n(\boldsymbol{R})|\nabla_{\boldsymbol{R}} n(\boldsymbol{R})\rangle \\ &\quad = \iiint \mathrm{d}^3\boldsymbol{r}' \, \Psi_n^*(\boldsymbol{r}' - \boldsymbol{R}) \{ -\mathrm{i}[\boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\mu}] \Psi_n(\boldsymbol{r}' - \boldsymbol{R}) + \nabla_{\boldsymbol{R}} \Psi_n(\boldsymbol{r}' - \boldsymbol{R}) \} \\ &\quad = \mathrm{i}\boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\mu}. \end{aligned} \tag{4}$$

Replacing (4) in (3), and taking account of the explicit form of the electric field due to an infinite charged line with linear density λ, we get

$$\gamma_n(C) = \oint_C \boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\mu} \; \mathrm{d}\boldsymbol{R} = \mu\lambda \tag{5}$$

which is just the AC phase.

Actually (as noted by Aharonov and Anandan for the analogous Berry derivation of the AB effect) the above derivation is gauge-dependent, and implies a cyclic evolution of the neutral particle which is absent in a real AC experiment.

Let us therefore consider the generalized gauge-invariant Berry phase (Yang phase)

$$\tilde{\gamma}_n(t) = \int_0^t \langle \Psi_n(t')| \left(\mathrm{i}\frac{\partial}{\partial t'} - e\hat{A}_0(t') \right) |\Psi_n(t')\rangle \,\mathrm{d}t' \tag{6}$$

first introduced by Yang in his gauge-invariant formulation of quantum mechanics [13] and further investigated by Kobe [14]. It holds in general for non-adiabatic and non-cyclic processes, reducing to the Aharonov–Anandan and Berry phase, respectively, in the corresponding special cases of cyclic, non-adiabatic and cyclic and adiabatic evolution [14].

In order to apply the Yang phase to the AC effect, we use the same procedure of Aharonov and Anandan for the AB effect. In other words, we consider the splitting and recombination of a beam of neutral particles around a charge line so that the system may be regarded as going backwards in time along the path of one beam and forwards in time along the other path. Moreover, it is assumed that two different Hamiltonians drive the system along the two paths [15]†. We assume that the two Hamiltonians have the form (1).

In this case, $A_0 = 0$ in (6), so that the Yang phase can be written as

$$\tilde{\gamma}_n(t) = \int_0^t \langle \Psi_n(t') | \hat{H}(t') | \Psi_n(t') \rangle \, dt'. \tag{7}$$

Assume now that the instants $t = 0$ and $t = \tau$ correspond to splitting and recombination of the neutral-particle beam. Then, the wavefunctions of the Hamiltonian (1) for the two beam paths C_l ($l = 1, 2$) are given by [6, 10]

$$\Psi_l(\boldsymbol{R}, \tau) = \exp\left\{-i \int_0^\tau E_l \, dt\right\} \exp\left\{i \int_{C_l} \boldsymbol{p} \, d\boldsymbol{R}\right\} \exp\left\{-i \int_{C_l} (\boldsymbol{E} \times \boldsymbol{\mu}) \, d\boldsymbol{R}\right\} \Psi_l(\boldsymbol{R}, 0). \tag{8}$$

Then, it follows from (7) that

$$\tilde{\gamma}_n(C) = \oint_C \boldsymbol{p} \, d\boldsymbol{R} + \oint (\boldsymbol{E} \times \boldsymbol{\mu}) \, d\boldsymbol{R} \tag{9}$$

where C is the closed curve union of C_1 and C_2.

The above expression is the gauge invariant Yang phase for the AC effect.

In conclusion, we have shown that the phaseshift produced by the AC effect is a special case of a geometrical phase of the Berry type. Clearly, an analogous result can be derived straightforwardly also for the more general case considered by Anandan [6] of a magnetic or electric dipole in a generic electromagnetic field.

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† Clearly, this can be done only under suitable assumptions, which are discussed in [15].